SARALA BIRLA PUBLIC SCHOOL

SARALA BIRLA PUBLIC SCHOOL (SARALA BIRLA GROUP OF SCHOOLS)

Birla Knowledge City, Mahilong, Ranchi

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Sub: Mathematics

Assignment-2

TRANSPOSE OF MATRIX

Transpose of matrix A: Let A be any matrix, if all row of A is change with their respective column then the obtained Matrix is called transpose of A. It is represented by A^T or A'.

$\Rightarrow$ If order of A is $m \times n$ then order of A^T will be $n \times m$

Property of transpose

a. $[A^T]^T = A$ *b.* $(A \pm B)^T = A^T \pm B^T$ *c.* $(kB^T) = kB^T$

Example : 5

$$A = \begin{bmatrix} 1 & 7 & 4 \\ -1 & -2 & 1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ 4 & 1 \end{bmatrix}$$

Example : 6 Order of Matrix A is 2×3 .what will be the order of A^T.

3×2

Questions

1. Find the transpose of the matrix $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

2. If A is a matrix of order 2×3 and B is a matrix of order 3×5 what is the order of the matrix $(AB)^T$

3. Find x, if $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}^T$

4. If $\begin{pmatrix} a+b & 2 \\ 5 & b \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 2 & 2 \end{pmatrix}^T$, then find *a* and *b.*

5. If $A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that : *a.* $(A+B)' = A' + B'$ *b.* $(A-B)' = A' - B'$

6. For the following matrices A and B, verify that $(A \cdot B)' = B' \cdot A'$ $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = [-1 \quad 2 \quad 1]$

7. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $A + A'$.

8. $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then show that $A + A' = I$

9. For the following matrices A and B, find $(AB)'$ and write order of resulting matrix. $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and

$B = [-1 \quad 2 \quad 1]$

10. If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, verify that

    a. $(A')'$ b. $(A + B)' = A' + B'$ c. $(kB') = kB'$, where k is any constant.

11. If any matrix $A = (1 \quad 2 \quad 3)$, write AA', where A' is transpose of A.

12. Find the value of $x, \ y, \ z$ if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

13. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then verified that $A^T A = I_2$

14. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then find the value of θ satisfy the equation $A^T + A = I$.

SYMMERTIC AND SKEW SYMMETRIC MATRIX

Symmetric matrix: A Matrix which is equal to their transpose called symmetric matrix.

i.e. $A' = A$ Example: $\begin{bmatrix} 1 & 5 \\ 5 & -2 \end{bmatrix}$; $\begin{bmatrix} 6 & 5 & 4 \\ 5 & 5 & -2 \\ 4 & -2 & 7 \end{bmatrix}$

Skew-Symmetric matrix: A Matrix which is equal to negative of their transpose called symmetric matrix.

i.e. $A' = -A$ Example: $\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 & 5 & 4 \\ -5 & 0 & 2 \\ -4 & -2 & 0 \end{bmatrix}$

Note: Symmetric and skew symmetric matrix is only possible in square matrix.

Property of symmetric and skew symmetric

- Is A be symmetric or skew symmetric then kA also be symmetric or skew symmetric matrix.
- Is A be any symmetric matrix and we add or subtract by constant in each element of A then result is also symmetric (It is not applicable for skew symmetric matrix)
- Sum or difference of two symmetric matrix is symmetric.
- Sum or difference of two skew symmetric matrix is skew symmetric.
- If A be any square matrix then

  $A + A^T = \text{symmetric matrix}$

  $A - A^T = \text{skew symmetric matrix}$
- Any square matrix can be express as a sum of symmetric or skew symmetric matrix by using the formula: $A = \left(\frac{A + A^T}{2}\right) + \left(\frac{A - A^T}{2}\right)$

1. Give an example of symmetric and skew symmetric matrix.
2. Write a square matrix of order 2×2 which is both symmetric and skew symmetric.

3. Show that the matrix $\begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is symmetric matrix.

4. Show that the matrix $\begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 1 \\ -5 & -1 & 0 \end{bmatrix}$ is skew symmetric matrix.

5. Show that matrix $\begin{bmatrix} 2 & -4 & 7 \\ -4 & 8 & 11 \\ 7 & 11 & 3 \end{bmatrix}$ is symmetric and $\begin{bmatrix} 0 & -4 & 8 \\ 4 & 0 & 1 \\ -8 & -1 & 0 \end{bmatrix}$ is skew symmetric.

6. For what value of *x*, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew – symmetric matrix.

7. If *A* and *B* are symmetric matrices, prove that $AB - BA$ skew – symmetric matrix.

8. If *A* and *B* are symmetric matrices of same order, then show that AB symmetric if and only if *A* and *B* commute, that is $AB = BA$.

9. Prove that the diagonal element of skew symmetric matrix is zero.

10. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 5 & 7 \end{bmatrix}$, verified that

    a. $A + A'$ is a symmetric matrix *b.* $A - A'$ is a skew symmetric matrix

11. Let A be any square matrix, then show that

    a. $A + A'$ is a symmetric matrix *b.* $A - A'$ is a skew symmetric matrix.

12. Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, as the sum of a symmetric and skew symmetric matrix.

13. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as a sum of symmetric and skew symmetric matrix.

14. Express the following matrix as the sum of a symmetric and skew symmetric matrix $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ and verify your result.

15. Show that the matrix $B'AB$ is symmetric and skew symmetric according as A is symmetric or skew symmetric.

DETERMINANTS

- Determinant is only possible corresponding to a square matrix.
- Determinants corresponding to a matrix A is written symbolically $|A|$.
- Nu. of rows or Nu. of columns is known as order of the determinants.
- Each determinants have fixed value which can be find by evaluating it.

Process to evaluate Determinants

- Determinants of order 2×2

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc \quad \text{Example} \quad \begin{vmatrix} 1 & 3 \\ 6 & 7 \end{vmatrix} = (7 - 18) = -11$$

- Determinants of order 1×1 ; $|4| = 4$

Minor and cofactor

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 5 & -3 \\ 4 & -3 & 3 \end{bmatrix}$$

▪ Determinants of order 3×3

We can evaluate determinants of 3×3 by the help of any rows and any column with sign of its position

i.e. $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$.

Example :6 Evaluate $\begin{matrix} + & - & + \end{matrix}$ $\begin{vmatrix} 3 & 2 & -5 \\ -4 & 1 & 3 \\ 0 & 6 & 7 \end{vmatrix}$

$\Rightarrow 3\begin{vmatrix} 1 & 3 \\ 6 & 7 \end{vmatrix} - 2\begin{vmatrix} -4 & 3 \\ 0 & 7 \end{vmatrix} + (-5)\begin{vmatrix} -4 & 1 \\ 0 & 6 \end{vmatrix}$

$\Rightarrow 3(7-18) - 2(-28-0) + (-5)(-24-0)$

$\Rightarrow -33 + 56 + 120$

$\Rightarrow 143$

__Minor__: Minor of any element is generally represented by the symbol M_{ij}.

$M_{12} = \begin{vmatrix} 5 & -3 \\ -3 & 3 \end{vmatrix} = (15-9) = 6$

$M_{33} = \begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} = (10-1) = 9$

__Cofactor__: minor with sign of its position is called cofactor. It is represented by symbolically C_{ij}.

$C_{ij} = M_{ij}$; when $i+j$ is even

$C_{ij} = -M_{ij}$; when $i+j$ odd

$C_{12} = -\begin{vmatrix} 5 & -3 \\ -3 & 3 \end{vmatrix} = -(15-9) = -6$

$C_{33} = \begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} = (10-1) = 9$

Question based on basic concept of determinant

1. If $A = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$, find $|A|$.

2. If $A = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, find the value of $3|A|$.

3. Evaluate : (i) $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$ (ii) $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ (iii) $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$

4. Evaluate : (i) $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$ (ii) $\begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

5. For what value of *x*, $\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix}$ is a singular matrix?

6. For what value of *a*, $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix?

7. Find the mirror of element 6 in the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$.

8. If A_{ij} is the factor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{ij} \cdot A_{ij}$

9. Find the minor and cofactors of all the elements of the determinant $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$

10. Using cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

11. Using cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

12. Find the cofactor of a_{12} in the following : $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

13. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n, find the value of $Det\,(A^n)$.

14. If $\begin{bmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\alpha \end{bmatrix} = \frac{1}{2}$, where α, β are the acute angle, then write the value of $\alpha + \beta$.

Properties of determinants

1. The value of determinants remain unchanged if its row transformed to their respective column. I.e. $|A| = |A'|$
2. If any two row or any two column are interchanged then value of determinants remain same but sign changes.
3. Any two rows or any two column of a determinant are identical the its value will be zero.
4. If all entries in any rows or column of a determinants are Zero then value of determinants will be zero.
5. Value of determinants corresponding to a skew symmetric matrix of order 3×3 is zero
6. Is all elements of any row or any column are as sum of two elements then they can be express as sum of two determinants following ways:-
$$\begin{vmatrix} x_1 + x_2 & a & p \\ y_1 + y_2 & b & q \\ z_1 + z_2 & c & r \end{vmatrix} = \begin{vmatrix} x_1 & a & p \\ y_1 & b & q \\ z_1 & c & r \end{vmatrix} + \begin{vmatrix} x_2 & a & p \\ y_2 & b & q \\ z_2 & c & r \end{vmatrix}$$
7. Product with scalar elements :
$$k\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = \begin{vmatrix} ka & kd & kg \\ kb & ke & kh \\ kc & kf & ki \end{vmatrix}$$
8. $|kA| = k^n |A|$ where n is order of determinants.

Question based on properties of determinant

1. By using properties of determinant evaluate

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

2. For $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, verify property I (the value of determinant remain unchanged if its rows and columns are interchanged)

3. If $\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$, then show that $|3A| = 27|A|$

4. Write the value of the determinant

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

5. The value of the determinant of a matrix A of order 3×3 is 4. Find the value of $|5A|$

6. If $A = \begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find the value of $|2A| + |3B|$

7. What is the value of $|3I_3|$, where I_3 is the identity matrix of order 3?