# SARALA BIRLA PUBLIC SCHOOL

Birla Knowledge City, Mahilong, Ranchi

CLASS-XII, (2020-21)

Sub: Maths

ASSIGNMENT - 1

# MATRICES

*MATRICES* :- Matrix is a collection of number arranged in a fixed number of Row or Column. It is simple method to represent any data.

*Applications* : it is used in solving linear equations, electronic spreadsheet program etc.

*General equation of matrix* $n \times n$ *order*

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & \cdots & a_{nn} \end{bmatrix}$$

*General equation of matrix* $3 \times 3$ *order*

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- A horizontal entries are call Rows.
- Vertical entries are called column.
- $a_{ij}$ is General element of $i^{th}$ Row and $j^{th}$ column.
- *Order of Matrix* : (No of Row) $\times$ (No of column)
- *Number of elements* : the product of nu. of Rows and nu. of columns gives number of element in the matrix. i.e. if any matrix have *n* rows and *m* column then the number of element in that matrix is *mn*.

Example 1 $\begin{bmatrix} 5 & \sqrt{3} & 6 \\ 5 & -6 & 3/2 \\ 2 & 1 & 0 \\ 1 & 5 & 6 \end{bmatrix}$

Nu. of rows $= 4$ Nu. of columns $= 3$

Order $= 4 \times 3$ Nu. of elements $= 12$

$a_{11} = 5$ (element of $1^{st}$ row and $1^{st}$ column)

$a_{32} = 1$ (element of $3^{rd}$ row and $2^{nd}$ column)

$a_{43} = 6$ (element of $4^{th}$ row and $3^{rd}$ column)

## QUESTION BASED OF BASIC CONCEPT OF MATRICES

1. In the matrix $\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write :

   *a.* The order of matrix, *b.* the number of elements, *c.* write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

2. $A = (a_{ij}) = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = (b_{ij}) = \begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$, then find $a_{ij} + b_{ij}$

3. Construct a $2 \times 2$ matrix, $A = [a_{ij}]$, whose element are given by: **a.** $a_{ij} = \dfrac{(i+j)^2}{2}$ **b.** $a_{ij} = \dfrac{(i+2j)^2}{4}$

4. Construct a $3\times 2$ matrix whose element are given by $a_{ij}=\frac{1}{2}|i-3j|$.

5. For a $2\times 2$, matrix $A=[a_{ij}]$ whose elements are given by v, write the value of $a_{12}$.

6. Write the element $a_{12}$ of the matrix $A=(a_{ij})_{2\times 2}$, whose elements $a_{ij}$ are given by $a_{ij}=e^{2ix}\sin jx$.

7. If the matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements.

8. If the matrix has 8 elements, what are the possible order it can have?

# Type of Matrices

1. ***Row Matrix*** :- Matrix which have only one row called Row Matrix.

$$[1 \quad 2 \quad 3] \; ; \quad [5 \quad -6 \quad 7 \quad 2]$$

2. Column Matrix :- Matrix which have only one column called Column Matrix.

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \; ; \quad \begin{bmatrix} 2/3 \\ -3 \\ 6 \\ 8 \end{bmatrix}$$

3. Square Matrix :- the matrix in which nu. of row is equal to nu. of column called square matrix.

$$\begin{bmatrix} 5 & 6 \\ 1 & -2 \end{bmatrix} \; ; \quad \begin{bmatrix} 2 & 5 & 6 \\ 1 & 2 & 3 \\ -4 & 7 & 6 \end{bmatrix}$$

4. Diagonal Matrix :- A square matrix in which only its diagonal element, element other than diagonal is zero called diagonal matrix.

$$\begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix} \; ; \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

5. Scalar Matrix :- In a diagonal matrix all its diagonal element are equal called diagonal matrix.

$$\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \; ; \quad \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

6. Identity matrix :- In a diagonal matrix all its diagonal element are unity(1), called Identity matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \; ; \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

# Equality of Matrix

Two matrix are equal if

i. They have same order

ii. And have same corresponding elements.

Note: given two matrices A and B are equal, then for solving equate corresponding element.

Example 2 $\begin{bmatrix} x+y & w+v & -5 \\ \sqrt{5} & x & v \end{bmatrix} = \begin{bmatrix} 12 & 15 & -5 \\ \sqrt{5} & 4 & 9 \end{bmatrix}$

Solution:

$x+y=12$, $x=4$

$\Rightarrow 4+y=12$

$\Rightarrow y=8$

$w+v=15$, $v=9$

$\Rightarrow w+9=12$

$\Rightarrow w=3$

# Questions

1. $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y--2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$. Find the value of *a, b, c, x, y* and *z*

2. Find the value of a, b, c and d from the given equation : $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$.

3. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x+y$.

4. Find the value of *a* if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 5 & 13 \end{bmatrix}$.

5. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of $\alpha$ is $A$ an identity matrix?

6. Write the number of all possible matrices of order $1\times 2$ with each entry *x* and *y.*

# Operations on Matrices

### *Addition & subtraction*

i. Two matrices can be added or subtracted if both have same order.

ii. Corresponding element is added. And obtained result have also same order.

Note : (i) $A+B = B+A$ (ii) $(A+B)+C = A+(B+C)$

(iii) $A \pm 0 = A$ (iv) $0 - A = A$

### *Multiplication with Constant*

$$A = \begin{bmatrix} 2 & 2/3 \\ 5 & 4 \\ -2 & 1 \end{bmatrix} \Rightarrow kA = \begin{bmatrix} 2k & 2/3k \\ 5k & 4k \\ -2k & 1k \end{bmatrix}$$

Example 3 $A = \begin{bmatrix} 7 & 7 \\ 4 & 6 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 10 \\ 3 & 5 \end{bmatrix}$, find :

$A+B$ and $A-B$

Solution:

$$A+B = \begin{bmatrix} 7 & 7 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 10 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 7 \\ 4 & 6 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 7 & 7 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 10 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 1 & 1 \end{bmatrix}$$

# Questions

1. Given $A = \begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix}$, find $A+B$.

2. $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$. Compute $2A-3B+4C$

3. Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

4. Find the matrix $X$ such that $2A+B+X=O$, where $A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$ & $B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$

5. If $3A-B=\begin{bmatrix}5 & 0\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}4 & 3\\ 2 & 4\end{bmatrix}$, then find the matrix $A$

6. If $A=\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}$ and $B=\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}$, then find the matrix $X$ such that $2A+3X=5B$.

7. Find the value of $X$ and $t$ if $2\begin{bmatrix}x & z\\ y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$

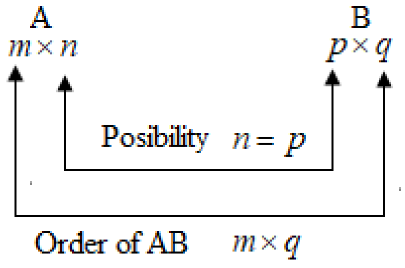
8. Find the value of $x+y$ from the following equation : $2\begin{bmatrix}x & 5\\ 7 & y-5\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$

9. Find matrix X and Y if, $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

# Multiplication of matrix

## *Possibility for Multiplication*

- $A_{m\times n}$ ; $B_{p\times q}$ product $AB$ is possible if $n=p$.
- And Result $AB$ have order $m\times q$




## *Properties of multiplication of matrices*

*i)* $A(BC)=(AB)C$

*ii)* $A(B+C)=AB+BC$

*iii)* $(A+B)C=AC+BC$

Note:

⇒ Product of matrices is not commutative i.e.

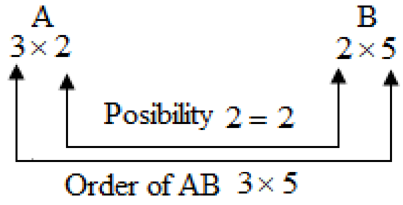
$AB\neq BA$

⇒ For matrix A and B $(A+B)\neq A^2+2AB+B^2$ because $AB\neq BA$. But if given $AB=BA$ then only apply the formula :

$(A+B)=A^2+2AB+B^2$

**Example 4 :** Two matrix A and B have order $3\times 2$ and $2\times 5$ respectively, then find order of AB and BA.

Solution:



# Questions

1. If *A* is the Matrix of order $3 \times 4$ and *B* is matrix of order $4 \times 3$, then find the order of *(AB)*..

2. Let $A = (a_{ij})_{m\times 3}$ ; $B = (b_{ij})_{p\times 4}$ and $C = (c_{ij})_{2\times 4}$ are such that $A_{m\times 3} \cdot B_{p\times 4} = C_{2\times 4}$ ; then find the value of m and p.

3. Find AB, if $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{bmatrix}$

4. Find AB and BA if $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -2 & 6 \\ 1 & 2 & 5 \end{bmatrix}$

5. If $A = [1 \quad -1 \quad 2 \quad 3]$ $B = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$, Find *AB* and *BA.*

6. Evaluate : $\left( \begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

7. Find $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$, Find $A^2$.

8. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify $A^2 = I$

9. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ show that $A^2 = O$

10. If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$ find $A^2$

11. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

12. Find x, if $[x \quad -5 \quad -1] \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

13. If $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find $A^{16}$

14. If the matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then find the value of $\lambda$.

15. Write the value of $x + y + z$ if $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

16. If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then write the value of *p.*

17. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $f(x).\ f(y) = f(x+y)$

18. Let $A = \begin{bmatrix} 0 & -\tan\frac{x}{2} \\ \tan\frac{x}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2 Show that :

$(I + A) = (I - A)\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

19. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ find a matrix D such that $CD - AB = O$

20. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$